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STRUCTURES OF INFLUENCE AND COOPERATION-CONFLICT

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This paper outlines a structural approach to the analysis of patterns of influence and amity-enmity in international politics. Structure is defined as a set of relations between pairs of social units, which justifies the use of graph theory to represent structures. Graph theoretical models of international structure are explored—one for each of the two types of relations. The author devises some typologies and observes changing structures. A tendency toward hierarchy in influence structures and toward bipolarization in cooperation-conflict structures is discussed. It is hypothesized that high degrees of hierarchy and polarization are inimical to international peace. The graph theoretical studies yield intuitively reasonable results and the models appear to be useful in making several concepts of international relations theory more precise.

INTRODUCTION

A structural approach can be of great value in interpreting or understanding political phenomena. By " structural," I mean primarily an approach which depends on binary relations—relations between pairs of objects. Branches of mathematics useful in dealing with relations are graph theory, matrix algebra, network theory, topology and lattice theory. Structural representations are consistent with the view that the social sciences must deal with a high degree of complexity in human affairs, that the units to be studied are something more than an aggregate of their component parts, and that things in general are interconnected—but in patterns which are not easy to discern with more traditional theoretical tools (see Boudon, 1968). The " structural " approach is consistent with the goals of such subdisciplines as gestalt theory in psychology, the structural anthropology of Levi-Strauss, sociometry, the structural linguistics of Noam Chomsky, and community power studies (of the Floyd Hunter variety) in political science.

The contemporary use of the words " structure " and " system " reflects a desire to deal with the complexity of international politics in a new way. Why not look at international politics as a pattern of relations between international actors? " State-centric " theorists stress relations between nation-states, while " transnationalists " stress those between states and other types of international actors, but both hold a common interest in the overall form of these relations. (Nye and Keohane, 1971, pp. 333–34). Yet, despite admirable attempts by Brams (1966), Galtung (1966; 1971), Bernstein and Weldon (1968), Caplow and Finsterbusch (1968) and Schofield (1971), to discover patterns in international relations, there is still a need for theoretical models of possible types of international structures.

One way of viewing international political structure is to look at influence and cooperation-conflict as relations. The normal way of dealing with these two concepts is to consider them to be " attributes." An attribute is a property or characteristic of a single social unit; a relation is a property of an ordered pair of social units. For example, if we say that nation X has more influence than nation Y, we are treating influence as an attribute (which might be indicated by military capability or diplomatic skill). If we say that nation X has influence over nation Y, then we are using influence in a relational sense. Thus, influence is a concept which may be either an attribute or a relation (or both). The same can be said of the concept of cooperation-conflict. The optimal policy would be to use attributive (cross-national or comparative) methods simultaneously with relational (structural) methods. But since the structural methods have been somewhat neglected, the main focus of this paper will be on structure.

There are, of course, alternative ways of looking at international structure. Other relational concepts such as " affective ties," " trust " or " communications " may be used. Other senses of the word structure may be used. For example, it is legitimate to define structure as a set of variables which are highly interdependent in a causal sense (as in causal models or econometrics) rather than as a set of actor-to-actor relations. But focusing on the relational sense of the words structure, influence, and cooperation-conflict is quite natural, given the current state of international relations theory. Event-interaction analysis, for example, has made it feasible to look closely at relations between pairs of states or actors. The analysis of transactions between pairs of nations—such as trade, student exchanges, migration, tourism and mail—is widely used in studying international relations. The structural notions of influence hierarchies, power blocs and polarization are frequently used in theoretical discussions of international systems. Thus, study of structure in the relational sense, and of influence and cooperation-conflict relations specifically, should contribute to our ability to observe international structures in general. It might also help to bring together some of the dangling threads of quantitative international relations theory.

STRUCTURES OF INFLUENCE

Influence here denotes a relationship between a pair of social units (individuals or aggregates of individuals), acting within a relatively specific domain, in which one unit is able, or potentially able, to change the behavior of the other in a desired direction (see Dahl, 1957, pp. 202–03). An *influence structure* is a set of units and influence relations (Emerson, 1962; Taylor, 1969, pp. 490–91). An *international influence structure* is an influence structure where the units are nation-states.

These definitions involve some strong, but not entirely unreasonable, assumptions:

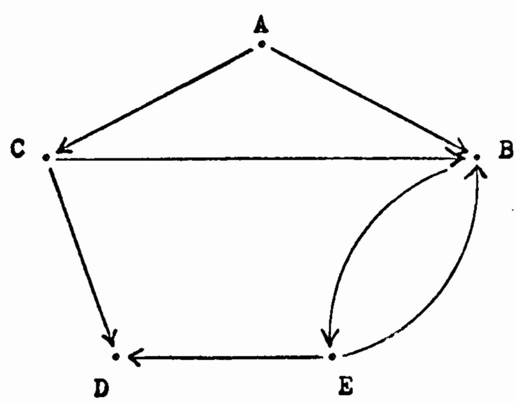
1) The units have well-defined boundaries and are internally cohesive. This may be appropriate for individuals, but is somewhat less appropriate for nation-states.

2) All influence relations involve pairs of units. Influence which may result from collective or multilateral action but which cannot be exercised in isolated pairs of units must therefore be ignored. Such influence frequently occurs in cohesive, small groups of individuals where there are established and consensual group standards and sometimes occurs in societies; but it is much rarer in international relations.

3) I will set aside many of the complicated conceptual issues connected with " power " and influence (e.g., the dispute over the observability of influence, as stated by Bachrach and Baratz (1962) in their theory of influence through " non-decisions ").

I hope the use of influence structures will serve to illuminate these sorts of conceptual problems in a new context.

By making one additional assumption—that influence relations either exist or do not exist and that they do not vary in intensity—it is possible to use directed graphs to represent influence structures and, thereby, make a large and growing body of mathematical concepts and theorems available to those who wish to study influence patterns. A directed graph, or *digraph*, consists of two basic elements—points (or nodes) and lines (or arrows). The points represent the social units in question and the lines, influence relations. Any digraph can be represented in two equivalent ways—by drawing



	A	B	C	D	E
A	0	1	1	0	0
B	0	0	0	0	1
C	0	1	0	1	0
D	0	0	0	0	0
E	0	1	0	1	0

FIGURE 1 Example of a digraph and its adjacency matrix

the graph or by specifying its adjacency matrix. The *adjacency matrix* consists of entries $\{a_{ij}\}$ such that $a_{ij} = 1$ if there is a line which goes from point i to point j and $a_{ij} = 0$ otherwise. Below is a digraph with five points and seven lines, and its adjacency matrix.

If there is a line from point A to point B, it is called AB. A *path* from point i to point j exists if there is any way to get from i to j by following lines and never retracing steps. For example, in Figure 1, a path from A to C is simply AC and a path from A to D is either AC, CD or AB, BE, ED. Note that EB, BE, ED is not a path from E to D since two of its lines originate from the same point, thus retracing a step; it is called a *sequence*. A *cycle* consists of a path together with a line from the terminal point of the path to the initial point. There is only one cycle in Figure 1—BE, EB.

A *semipath* is a sequence of lines that would be a path except that one or more lines point in the wrong direction. CB, AB is a semipath from C to A in Figure 1. A *semicycle* consists of a semipath together with a line between the initial and terminal points. The *length* of a sequence, path, semipath or cycle is the number of lines in it. The *distance* from point i to point j is the length of the shortest path from i to j. If there is a path from i to j, j is said to be *reachable* from i. If no such path exists, j is *unreachable* from i and the distance from i to j is considered to be infinite. Thus, there are two additional matrices associated with each digraph: the *distance matrix* with entries $\{d_{ij}\}$ where d_{ij} is the distance from i to j if $i \neq j$ and $d_{ij} = 0$; the reachability matrix with entries $\{r_{ij}\}$ where $r_{ij} = 1$ if $d_{ij} \neq \infty$ and $r_{ij} = 0$ if $d_{ij} = \infty$. The distance and reachability matrices for Figure 1 are given below.

Distance Matrix

	A	B	C	D	E
A	0	1	1	2	2
B	∞	0	∞	2	1
C	∞	1	0	1	2
D	∞	∞	∞	0	∞
E	∞	1	∞	1	0

Reachability Matrix

	A	B	C	D	E
A	1	1	1	1	1
B	0	1	0	1	1
C	0	1	1	1	1
D	0	0	0	0	0
E	0	1	0	1	1

When a given point cannot reach any other point in the graph it is either an *isolate* (a point which is both unreachable from all other points and unable to reach any other points) or a *receiver* (a point unable to reach other points but reachable from at least one other point). Point D in Figure 1 is a receiver. A point is a *source* if it can reach all the other points in the digraph. Point A in Figure 1 is a source.

A digraph is strongly connected or *strong* if every point can reach every other point in the digraph. That is, there is a path between every pair of points. A digraph is unilaterally connected, or *unilateral*, if for any given pair of points at least one is reachable from the other. A digraph is *strictly unilateral* if it is unilateral but not strong. A digraph is weakly connected, or *weak*, if every pair of points is joined by a semipath. A digraph is *strictly weak* if it is weak but not unilateral.

A *subgraph* of a digraph contains a subset of the nodes in the digraph and some or all of the lines that joined them. A *maximal subgraph* is a subgraph which has all the lines that connected the points in the original digraph. A subgraph is said to be maximal with respect to a certain property if there is no larger subgraph that contains it as a subgraph and has the desired property. A *strong component* of a digraph is a maximally stong subgraph. A *weak component* of a digraph is a maximally weak subgraph. Since the digraph in Figure 1 is strictly unilateral, the only weak component is the digraph itself. The only strong component with more than one point is the one that contains points B and E. A digraph is *disconnected* if it has more than weak component. When this is the case, it is usually necessary or desirable to analyze each weak component separately.[1]

The lines in digraphs representing influence structures stand for a *direct* influence relation. A → B means that " A has direct influence over B," " B is directly influenced by A," " B is directly subordinate to A," " A is directly superordinate to B," or " B is dependent on A." A common assumption in graph theoretical treatments of influence structures is that of antisymmetry—that is, if A directly influences B, then B cannot directly influence A. Such structures are called *dominance structures* here, and correspond to a class of digraphs called " tournaments " (Bartos, 1967, pp. 50–51). An example of a study of international dominance structures is Caplow and Finsterbusch (1968) which focused on the balance of payments relation. The assumption of antisymmetry will not be used here. Instead, it will be said that if A ←→ B, then A and B are " mutually influential " or " interdependent." For most domains of action in international relations, some mutual influence or interdependence exists.

Two approaches will be used in analyzing influence structures below: (1) the identification of types of structures, and (2) the derivation of local properties of units (or points) from the structural pattern of influence. Five main types of influence structures will be discussed: trees, semi-lattices, strictly acyclic structures, " hishinters " (defined below) and cliques. Each will be defined formally, and the relation between types of structures and two measures of local properties will be explored.

Trees

Perhaps the most familiar type of influence structure is the " tree from a point " or, more simply, the *tree*. In graph theoretical terms, a tree is a digraph with no semicycles (Cartwright, et al., 1965, p. 409). But there is a more intuitive definition—a tree is an influence structure in which no unit has more than one direct superordinate and in which no member is the direct or indirect subordinate of his own subordinate. It follows from both definitions that each tree has a point which has no superordinates (Friedell, 1967, p. 47). This point is a unique source. There are many kinds of trees. For example, four different trees are illustrated in Figure 2.

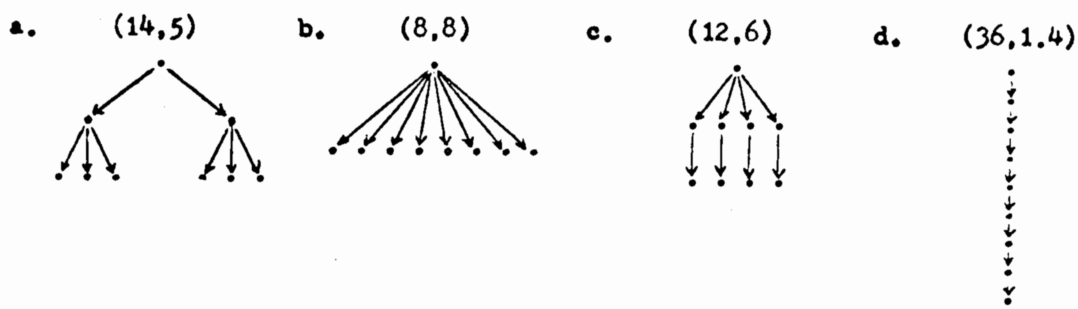


FIGURE 2 Examples of trees with nine points.

The number of *levels* in a tree equals one plus the length of the longest path (or chain of command). Thus, Figure 2d has nine levels, Figures 2a and 2c have three levels, and Figure 2b has two levels. The *status* of a point in a tree has been defined by Frank Harary (1959) to be

$$(1) \quad \text{status}_i = \sum d_{ij} \qquad \text{for all finite } d_{ij}.$$

Thus, the more insulated a superordinate is from his subordinates—whether by the number of levels or by having very few direct subordinates—the higher is his status. This notion was inspired by C. Northcote Parkinson's " humorous " law of bureaucratic behavior. The status for the source of each tree in Figure 2 is given as the first number in parentheses next to the source. Status for the source is maximal in trees like that in Figure 2d (a simple path) and minimal for trees like that in Figure 2b (sometimes called a " star ").

The *control* of a point in a tree is defined in the following

$$(2) \quad \text{control}_i = \sum_{j \neq i} \frac{1}{d_{ij}}$$

This means that a superordinate has the greatest control over his direct subordinates and that control decreases as a function of the length of the chain of command. The control for the source of each tree in Figure 2 is the second number in parentheses next to the source. Note that the status of the sources is inversely related to their control. Note also that the status and control of any point on a given level in Figure 2 exceeds or equals that of its subordinates on lower levels. Thus, trees differ in the number of levels they have, in the status and control of their sources and in the distribution of status and control among the other points.

A real world example of a tree is the bureaucratic structure of an organization. In this case, the points in the tree would correspond to individuals, roles or offices. It is true, of course, that the informal pattern of influence in a bureaucracy does not always resemble its organizational chart. But a tree-shaped pattern often represents the expressed ideal. Some scholars have suggested that some imperial systems, such as that composed of Spain and its American colonies in the 15th and 16th centuries, were trees. In such systems, the points correspond to colonies and colonial powers. Such trees are thought to be more likely to resemble Figure 2b than the other trees in Figure 2 (Merritt, 1963; Galtung, 1971). When international relations theorists speak of unipolar international systems or subsystems, they are usually thinking in terms of tree-shaped pattern of influence for a set of nation-states.

Semilattice

A less familiar type of influence structure is the *semilattice*, a digraph with no cycles and a unique source. Intuitively, a semilattice is a structure in which no unit is the subordinate of his own subordinates. Thus, a tree is a special kind of semilattice. Henceforth when discussing semilattices I will be referring to semilattices which are not trees, or *strict semilattices*. In a semilattice, a unit may have more than one direct superordinate (unlike units in trees). Just as there are many kinds of trees, there are also many kinds of semilattices.

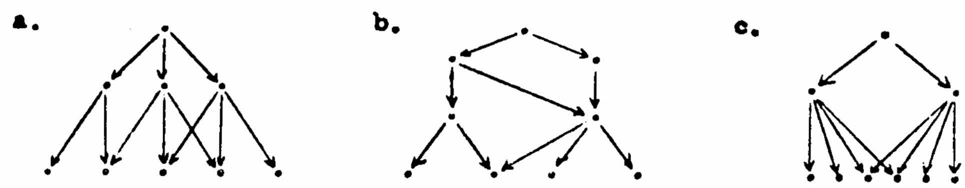


FIGURE 3 Examples of semilattices with nine points.

In order to adapt the definitions of status and control to semilattices one must decide whether a given unit can be counted as an indirect subordinate of another unit more than once. For the present, I will assume that double counting exaggerates the level of control or status of the unit in question. Given this assumption, if one compares the distribution of status and control in a tree which is formed from the semilattice by removing a sufficient number of lines, one will find that the semilattice allows units at intermediate levels of the structure to have more control and status than they might have in a comparable tree.[2]

Strict semilattices, like trees, have different levels and different distributions of status and control. But unlike trees, they allow a unit to have more than one direct superordinate. This increases the possibility that the unit will get conflicting orders, especially if the most influential unit, the source, allows units at intermediate levels to exercise their own discretion in certain matters. Semilattices also allow for greater control and status of intermediate units—sometimes referred to as " decentralization." Morris Friedell (1967, p. 48) suggests that an influence structure racked by conflicting orders will more likely resemble a tree than a semilattice. Similarly, a rigidly hierarchical but relatively decentralized structure will more likely resemble semilattices than trees.

The influence structure of a bureaucracy may be a semilattice. In certain corporations, the auditing or quality control divisions may have authority over all other subdivisions. Thus, each subdivision may have two superordinates—their own division and the auditing or quality control division. Another semilattice, the federalist political system, has a unique source of authority (e.g., the federal government which exerts influence over political units at different levels, such as regional, state, county, metropolitan and city governments) through a variety of influence channels (see Alexander, 1965). The structure of influence among certain bodies in the United Nations resembles a semilattice in that the specialized agencies have overlapping clienteles and jurisdictions while the Secretary General acts as the supreme authority. Semilattices may occur within empires, for example, when colonies begin to create their own colonies and have to settle for shared influence when spheres of influence overlap (see Figure 3a).

Acyclic Diagraphs

Both trees and semilattices are *acyclic* digraphs—that is, they have no cycles. The so-called *strictly acyclic* digraphs, unlike trees or semilattices, may have more than one source or more than one *transmitter* (a point from which lines originate but to which none are directed). All three digraphs have clearly defined levels of hierarchy.

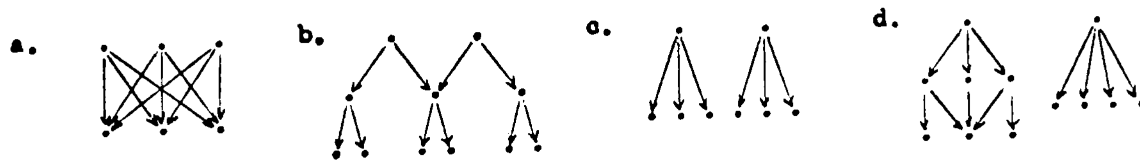


FIGURE 4 Examples of strictly acyclic structures.

Strictly acyclic structures may be like trees in that no member has more than one superordinate (as in Figure 4c). In such cases, the strictly acyclic structure is a set of disconnected trees. A strictly acyclic structure may be a set of disconnected semilattices or a set of disconnected trees and semilattices (as in Figure 4d). Strictly acyclic structures, therefore, can be connected (as in Figures 4a and 4b) or disconnected. Like trees and semilattices, they may differ in the number of levels and the distribution of status and control.

In Figure 4a, the three sources can directly influence all the other points. This may be the case in a business partnership (where the partners are in conflict), in a nation with shared or rotating leadership or in an international system in which the leading nations have totally overlapping empires or spheres of influence. In Figure 4b, each of the sources has a joint and a discrete subordinate—like wo empires which share influence over a third, weaker empire (e.g., Turkey at the end of the 19th century). In Figures 4c and 4d the transmitters' spheres of influence do not overlap (cf. an international system with a pair of empires, an organization with parallel but non-overlapping bureaucratic structures, or a nation with a pair of disconnected governmental authority structures).

All acyclic structures have clearly defined levels, but a particular level does not contain a point unless every path between it and a source or transmitter is of equal length. For example, does point a in the tree in Figure 5a belong in the second or third level of the structure? And does point b in Figure 5b (a semilattice) belong in the second or third level.

All acyclic structures are *hierarchical* in the sense that they are layered and have consistent chains of command. All acyclic structures are also antisymmetric—there is no interdependence or mutual influence in them. But there are also structures which are hierarchical and which have some interdependent units (see Boyle, 1969). I have dubbed them *hishinters*—short for hierarchical influence structures with horizontal interdependence. In a hishinter, only units which occupy the same level of the structure are interdependent. The *underlying structure* of an influence structure is defined as the digraph which results from the removal of all symmetric lines. The underlying structure of a hishinter is always an acyclic structure. If there is a cycle of length greater than two in a hishinter, the maximal subgraph containing the points in the cycle is always symmetric.

The underlying structure of a hishinter may be a tree (Figure 6a), a semilattice (Figure 6c) or a strictly acyclic structure (Figures 6b and 6d). The level of a unit in a hishinter refers to the level it would have in the underlying structure. The status and control of a unit is enhanced if the number of its subordinates is effectively increased as a result of its interdependence with another unit—but the previously defined measures of status and control are no

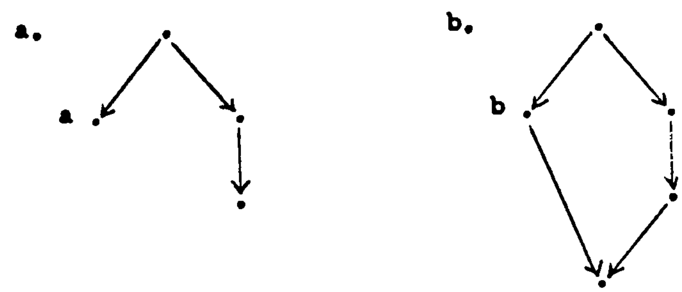


FIGURE 5 Examples of acyclic structures with ambiguous level assignments.

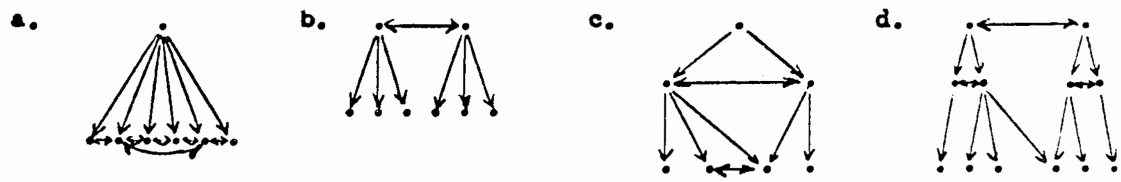


FIGURE 6 Some hishinters.

longer applicable. A unit's status or control is simultaneously increased and decreased when it becomes mutually dependent with another unit. I will assume that the distance from one member of a structure to another equals zero if the two are interdependent and that the distance to all other points in the structure will be the same for both. Thus, if all the units of a structure on a given level are interdependent, they will all have the same degree of status and control. Status and control measures, appropriately modified for use with hishinters, are as follows:

$$(3)\quad \text{status}^1{}_i = \sum_{j \neq i} d^1{}_{ij} \qquad \text{where } d^1{}_{ij} = d_{ij}a_{ij}a_{ji}$$

$$(4)\quad \text{control}^1{}_i = \sum_{j \neq i} i/d^1{}_{ij} \qquad \text{and if } d^1{}_{ij} = 0,\ i \neq j, \text{ then } d^1{}_{ik} = d^1{}_{jk} \text{ for all } k$$

In the real world, a hishinter organization might combine interdepartmental decision-making committees on higher levels and highly integrated work groups on lower levels. A national analogue would be a federal system in which the smaller governmental units form coalitions (for lobbying purposes or whatever) in order to improve their positions *vis-à-vis* larger units—as in the contemporary League of Mayors in the United States (see Figure 6a). On the international level, an analogue would be the formation of coalitions by colonies or developing nations (as in the Organization of Petroleum Exporting Countries, (OPEC) to improve their bargaining positions with respect to major powers.

Hishinters tend toward interdependence between units on higher rather than lower levels. Why? Patrick Doreian (1970, p. 9?) suggests that the more complex the task, the greater the need for coordination and, hence, interdependence. And the higher-level units usually tackle the most complex tasks. Similar reasoning led Johan Galtung (1966; 1971) to hypothesize that interaction and interdependence increase with the total rank of pairs of individuals or nations. Galtung sees the contemporary international influence structure as a hishinter with interdependence only at the higher levels (as in Figure 6b).

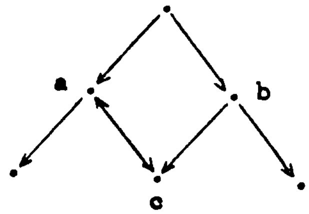
Most of the definitions of polarization in international systems can be restated in terms of hierarchical influence structures. If alignment precedes influence, poles are stratified, and a source or transmitter in a hierarchical influence structure is analogous to a pole of bloc leader in international systems, then an international system is tightly polarized if it has a disconnected influence structure and the *number of poles* equals the number of sources or transmitters. Thus Figures 4b, 6b, and 6d could be influence structures for loosely bipolarized international system; Figures 4c and 4d could be structures corresponding to tightly bipolarized systems; and Figure 4a could be a loosely multipolarized system. With structures like that in Figure 6b, it may be necessary to say that although there are two sources the interdependence between pole leaders (resulting from a *détente*, for example) suggests that the system is *latently* unipolar.

Vertical interdependence in a hierarchical structure will tend to blur the levels of the system. If a particular unit interacts with two units on a higher level—as an equal in one case and subordinate in another—its level will be difficult to determine, even when the underlying structure has unambiguous level assignments (see Figure 7).

A similar problem arises in structures with no interdependence but with cyclical patterns of influence. The simplest cycle of influence is illustrated in Figure 8.

One solution could assign all units in the same strong component of a digraph to the same level. Thus, a and c in Figure 7 and d, e, and f in Figure 8 would be assigned to the same levels. But this solution fails to differentiate between symmetric and antisymmetric structures (compare Figure 8 to Figure 9a).

Thus, influence structures which have either vertical interdependence or cycles of influence (of length greater than two) will be considered



Does c belong on the second level or the third? Does b belong on a higher level than a?

FIGURE 7 An influence structure with vertical interdependence.

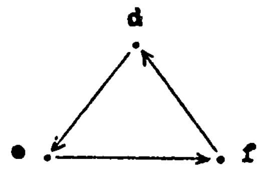


FIGURE 8 An influence structure with a cycle of influence.

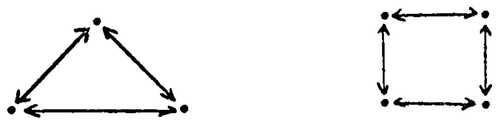
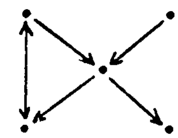


FIGURE 9 Symmetrical cycles of influence.

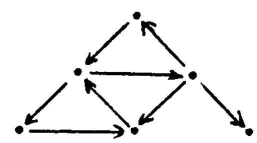
nonhierarchical structures due to hazy level assignments. Nonhierarchical structures also include structures with both vertical interdependence *and* cycles of influence as well as all perfectly symmetric structures. The most important subtype of nonhierarchical structure is the *clique* or complete, symmetric digraph. Below are some examples of these nonhierarchical influence structures.

Vertical interdependence can appear in an organization when the complex bureaucratic structure hinders efficient coordination of subordinates. In such bureaucracies, upper level superordinates will be tempted to form "partnerships" with lower level subordinates in order to get what they want without going through bureaucratic channels. The by-passed middle-level bureaucrats then begin to fear for their status and control. Such threatened individuals may criticize the resulting anarchy and argue for the original hierarchy.

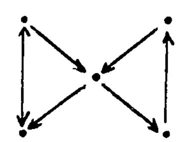
Cycles of influence can occur among nations. One country might depend for titanium ore on another, which in turn might depend for iridium on another, which in turn depends on the bauxite exports of the first country. Cyclical patterns of



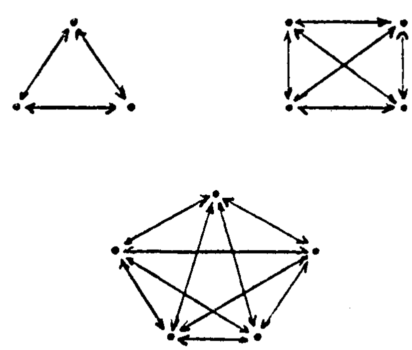
(a) vertical interdependence (see also Figure 7).



(b) asymmetrical cycles of influence (see also Figure 8).



(c) vertical interdependence *and* cycles of influence.



(d) cliques of various sizes.

FIGURE 10 Nonhierarchical influence structures: subtypes.

dependency in international trade, a result of balance of payments deficits and surpluses, foster fears and anxiety. Perfectly symmetric influence structures have no levels, and all the units are sources (assuming that the structure is connected). In cliques, all units have the same amount of status and control. Thus, cliques are equalitarian structures. Committees or small decision-making groups sometimes assume this form, especially those which operate on a " sense of the meeting " basis rather than on majority rule. Committees with majority rule and *stable* majority coalitions more closely resemble hishinters than cliques since the majority coalition may be considered to occupy a higher level than the minority coalition. Committees with *shifting* coalitions, however, may resemble cliques if every member belongs to at least one majority coalition.

Cliques such as the League of Mayors and the European Economic Community act as subordinate parts of international influence.

Figure 11 is a graph of inclusion relations among the types of influence structures discussed above. A line from type A to type B means that type B is a special case of type A. Such graphs are sometimes called topology trees.

Status and control are measures of *local* properties of influence structures—that is, they tell you something about the unit as a result of its position in the structure. I intend to use at least two other measures of local properties in analyzing international influence structures. The first is centrality:

$$\text{(5)} \qquad \text{centrality}_i = \frac{\sum_{i,j} d_{ij}}{\sum d_{ij}} \qquad \text{for all finite } d_{ij}.$$

Centrality, like control, measures the ease with which a unit can influence all the other units, assuming that the ease of influence decreases with distance (Bavelas, 1950), and is inversely related to status. However, centrality and status relate to each other.

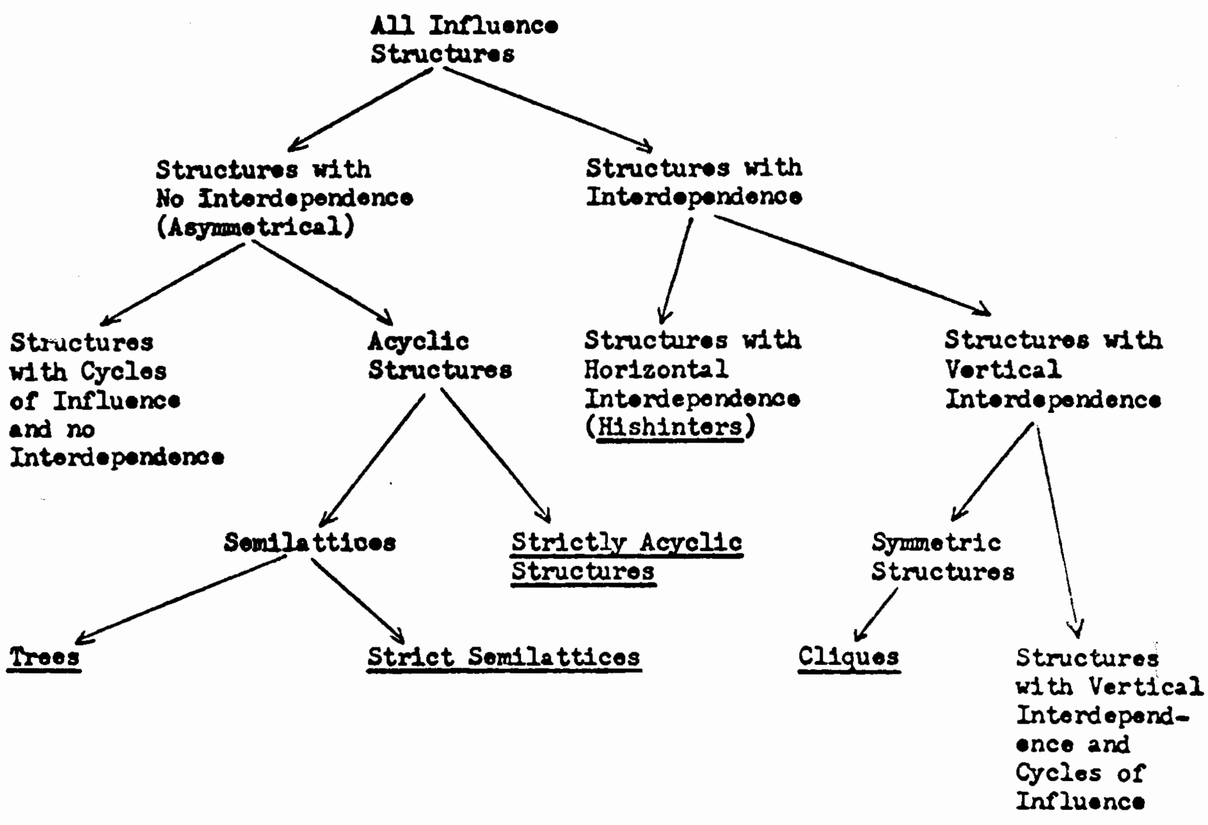


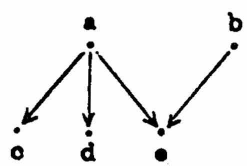
FIGURE 11 A typology tree of types of influence structures.

$$(6)\quad \text{centrality}_i = \frac{c}{\text{status}_i} \quad \text{where } c = \sum_{i,j} d_{ij}$$

Centrality was designed to be used in connection with symmetric, communication structures and does not give intuitively valid results when used with antisymmetric structures (see Figure 12). In addition, it is undefined for receivers and isolates. Several improved measures of centrality have been suggested (Beauchamp, 1965; MacKenzie, 1966; Sabidussi, 1966) but none of these completely eliminates all the drawbacks of the measure.

as outgoing sequences of influence. While measures of status and control cannot discriminate between receivers and isolates or between receivers with few incoming paths and sequences and receivers with many incoming paths and sequences, Taylor's measure can. In addition, the measure was designed to discriminate between structures with asymmetric and symmetric cycles of influence.

I have applied these measures to the study of influence structures resulting from import asymmetries in the Western Hemisphere. In these structures, I assume that a line of influence from



Distance Matrix

	a	b	c	d	e	centrality	status*
a	0	∞	1	1	1	4/3	3
b	∞	0	∞	∞	1	4	1
c	∞	∞	0	∞	∞	undefined	0
d	∞	∞	∞	0	∞	"	0
e	∞	∞	∞	∞	0	"	0

* For this particular diagraph, the measure for status equals those for both control and Taylor's influence.

FIGURE 12 An antisymmetric digraph and the centrality measure.

Another measure of local properties that I intend to use was devised by Michael Taylor (1968). This measure assumes that " influence " is a function of the number of units that can be influenced by the unit in question at a given distance as well as the number of units that can influence the unit in question at a given distance. To compute the measure, we first compute the powers of the adjacency matrix, A, A^2, A^3, . . . , A^n. The entries of the matrix, A^n, are equal to the number of sequences from i to j of length equal to n. Taylor defines his index of structural influence to be

$$(7)\quad \text{influence}_i = \sum_{n=1}^{\infty} w_n \frac{\dfrac{r_i(n) - c_i(n)}{N}}{\sum_{j=1} r_j(n)}$$

where N = the number of units in the structure
$w_n = (1/2)^{n-1}$
$r_i(n)$ = the number of sequences from i to other nodes of length $= n$
= the i^{th} row sum of the matrix A^n
$c_i(n)$ = the number of sequences from other nodes to i of length $= n$
= the i^{th} column sum of the matrix A^n

This particular measure considers sequences as well as paths of indirect influence and incoming as well

nation i to nation j exists if more than 1 percent of nation j's imports originate in nation i. Following the examples of Bernstein and Weldon (1968) and Schofield (1971) I used the United Nations' *Yearbook of International Trade Statistics* in order to derive import influence adjacency matrices for 1958 and 1968 (see Tables I and II).

This trade relationship may be thought of as an influence relationship for the following reasons:

1) j's economy depends to a certain extent on continuing imports from i,

2) i can threaten j with higher prices or an embargo,

3) the fact that j receives imports from i means that private or official representatives from i may be present in j and may put direct pressure on i if the trade relation is threatened.

Nation j cannot realistically counter the threats or pressure of nation i in this particular domain of influence unless it also contributes a significant share of nation i's imports. It may, of course, have other means of influence. The figure, 1 percent, is a relatively arbitrary cutoff point which, in later studies, will be supplemented by lower and higher cutoffs. Unlike some previous studies (such as Brams, 1966), we use unadjusted values of imports without controlling for the size or wealth of the

TABLE I

Adjacency matrix for the import influence structure, Western Hemisphere according to the United Nations' *Yearbook of International Trade Statistics*, 1958

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1. Argentina	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
2. Bolivia	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3. Brazil	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
4. Guyana	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5. Canada	0	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
6. Chile	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0
7. Colombia	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
8. Costa Rica	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9. Cuba	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
10. Dom. Rep.[a]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11. Ecuador	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12. El Salvador	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
13. Guatemala	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
14. Haiti	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15. Honduras	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
16. Mexico	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	0	1	0	0	0	1	0	0
17. Nicaragua	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18. Panama	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0
19. Paraguay	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20. Peru	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
21. United States	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
22. Uruguay	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
23. Venezuela	1	0	1	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	1	1	0

[a]Dominican Republic.

TABLE II

Adjacency matrix for the import influence structure, Western Hemisphere, according to the United Nations' *Yearbook of International Trade Statistics*, 1968

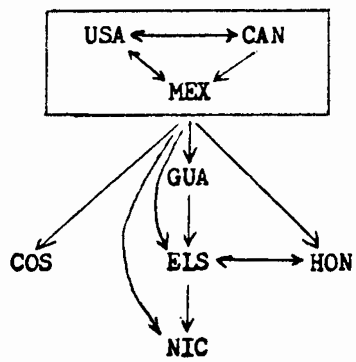
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1. Argentina	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0
2. Bolivia	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3. Brazil	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0
4. British Guiana	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5. Canada	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	0	1	0	1	1	1	1
6. Chile	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
7. Colombia	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	0	1	0
8. Costa Rica	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	1	1	0	0	0	0	0
9. Cuba	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10. Dom. Rep.[a]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11. Ecuador	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12. El Salvador	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0
13. Guatemala	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0
14. Haiti	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15. Honduras	0	0	0	0	0	0	0	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0
16. Mexico	0	0	0	0	0	0	1	1	0	0	0	1	1	0	1	0	1	0	0	0	1	1	0
17. Nicaragua	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
18. Panama	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0
19. Paraguay	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
20. Peru	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21. United States	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1
22. Uruguay	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23. Venezuela	1	0	1	0	1	1	0	1	0	1	1	1	0	0	1	0	1	1	0	1	1	1	0

[a]Dominican Republic.

different nations. Controls are consciously omitted so that correlation between economic size and local measures of influence can be computed in later studies.

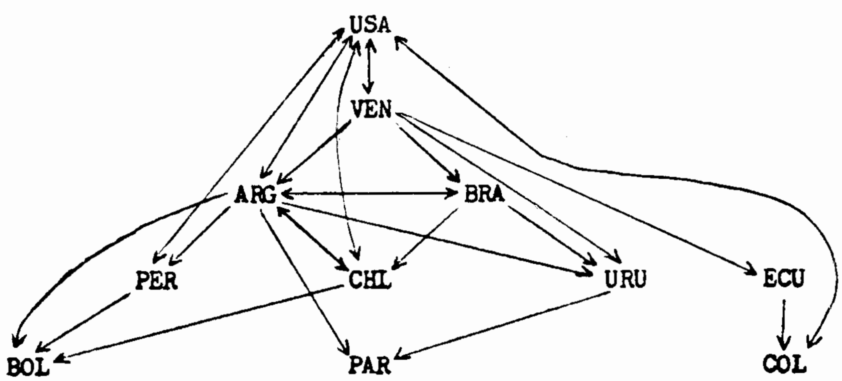
The graphs of substructures of import influence in the Western Hemisphere are given in Figures 13 and 14. In 1958, the import influence substructure for the North and Central American nations was almost a perfect hishinter of the Galtung variety with four levels (see Figure 13a). As one might expect, the United States, Canada and Mexico occupy the top level (in a slightly imperfect clique). Nicaragua occupies the lowest level. In 1968, the North and Central American substructure still resembles a hishinter. But by this time, the Central American nations have almost formed a clique—thus reducing the number of levels to two (see Figure 14a). The improved position of the Central American nations—largely due, one supposes, to the relative success of the Central American Common Market in increasing regional trade—reflects higher scores on the measure of control (see Table III). With the Central American nations still subordinate to their bigger neighbors to the North, scores on Taylor's measure of influence remain approximately the same. Since the structure is neither a perfect symmetry or a tree, the measures of status and centrality reveal relatively little.

Import influence substructures for the United States and South America remained relatively unchanged from 1958 to 1968 (see Figures 13b and 14b). This was true despite the formation of the Latin American Free Trade Association in the



Each member of this subset has direct import influence over each Central American state.

(a) North and Central America.



The United States has direct import influence over all South American states but only mutual influence relations are shown for the sake of simplicity.

(b) The United States and South America.

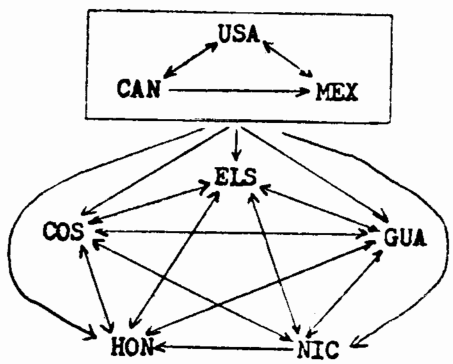
Abbreviations Used: ARG = Argentina; BOL = Bolivia; BRA = Brazil; CAN = Canada; CHL = Chile; COL = Columbia; COS = Costa Rica; ECU = Ecuador; ELS = El Salvador; GUA = Guatemala; HON = Honduras; MEX = Mexico; NIC = Nicaragua; PAR = Paraguay; PER = Peru; URU = Uruguay; USA = United States; and VEN = Venezuela.

FIGURE 13 Import influence substructures, Western Hemisphere, 1958.

interim. The relatively hierarchical substructures showed some vertical interdependence (the United States). Venezuela occupied a position of relatively high control and influence—probably as a result of its oil exports. Bolivia, Paraguay and Uraguay occupied lower levels of the substructure. Changes in economic performance, as in Bolivia and Uraguay, reflect changes in the measures of influence and control (see Table III). Again, the measures of status and centrality do not reveal much about the structure.

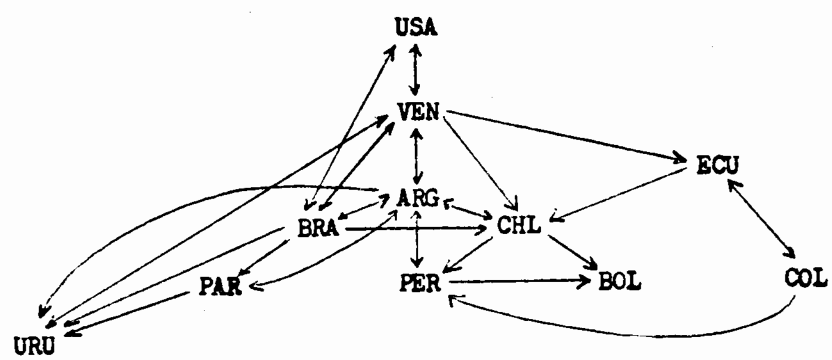
Common sense bears out these results, which may shed light on the debates about Latin American integration efforts. Beyond that, they suggest a connection between a nation's position in an international influence structure and its level or rate of development. This, in turn, may affect a nation's alignments in international politics. In 1968, for example, Cuba no longer depended heavily on other countries in the Western Hemisphere (but also had sacrificed its control) when it altered its international posture. The options open to a relatively dependent nation—to cut ties to other nations, diversify exports, diversify trading partners, or join a regional economic organization—relates closely to the structural effects of these policies. Cuba traded dependency on the United States for dependency on the Soviet Union, while the Central American states traded increased dependency on each other for decreased dependency on the United States, Canada, and Mexico. The results also seem to confirm Latin America's view of itself as the victim of a centralized and hierarchical trade structure. They suggest that less developed nations can alter their economic status by attempting to change the structure of international trade.

The results can also help to determine the effect of setting the cutoff for import influence relations



Each member of this subset has direct import influence over the Central American states except that Mexico does not have direct influence over Costa Rica.

(a) North and Central America.



Abbreviations: Same as in Figure 13.

(b) The United States and South America.

FIGURE 14 Import influence substructures, Western Hemisphere, 1968.

TABLE III
Local measures of influence, Western Hemisphere, 1958 and 1968

Nation	Status 1958	Status 1968	Control 1958	Control 1968	Influence 1958	Influence 1968	Centrality 1958	Centrality 1968
1. Argentina	37	46	14.5	12.2	0.03	—0.01	15.6	12.2
2. Bolivia	0	0	0	0	—0.06	—0.05	0[a]	0
3. Brazil	40	36	13.0	13.5	0.02	0.03	14.4	15.5
4. Guyana	0	0	0	0	—0.03	—0.01	0	0
5. Canada	25	25	28.5	19.0	0.16	0.15	23.1	22.4
6. Chile	40	64	13.0	8.8	0.01	—0.02	14.4	8.8
7. Colombia	43	59	11.5	10.1	0.00	—0.01	13.4	9.5
8. Costa Rica	0	7	0	5.5	—0.03	—0.05	0	79.9
9. Cuba	42	0	12.0	0	0.01	0.00	13.7	0
10. Dominican Republic	0	0	0	0	—0.05	—0.03	0	0
11. Ecuador	63	67	8.2	8.2	—0.02	—0.01	9.2	8.3
12. El Salvador	2	8	2.0	5.0	—0.05	—0.06	288.5	69.9
13. Guatemala	5	8	2.0	5.0	—0.03	—0.04	115.4	69.9
14. Haiti	0	0	0	0	—0.03	—0.01	0	0
15. Honduras	3	8	1.5	5.0	—0.05	—0.04	192.3	69.9
16. Mexico	38	34	14.0	14.5	0.04	0.07	15.2	16.4
17. Nicaragua	0	10	0	4.3	—0.06	—0.05	0	55.9
18. Panama	3	8	3.0	5.0	—0.01	—0.01	192.3	69.9
19. Paraguay	58	65	9.0	8.2	—0.03	—0.01	10.0	8.6
20. Peru	42	65	12.0	8.2	0.00	—0.04	13.7	8.6
21. United States	22	21	22.0	21.0	0.13	0.16	26.2	26.6
22. Uruguay	18	0	6.9	0	—0.05	—0.06	7.4	0
23. Venezuela	36	28	15.0	17.5	0.09	0.13	16.0	20.0

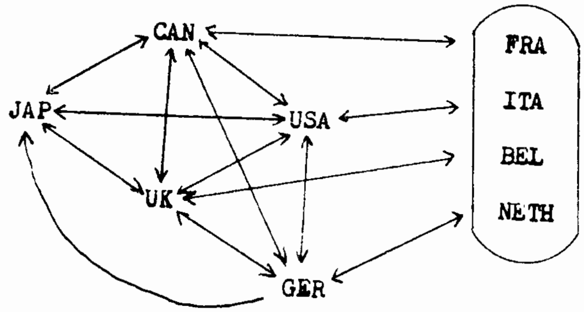
[a] A centrality score of zero is equivalent to an " undefined " score.

at one percent of total imports. A high cutoff level for the Western Hemisphere would result in a star-shaped tree (like Figure 2b) whose source is the United States. The United States would provide a large proportion of the imports of most Western Hemisphere nations, but not vice versa. A cutoff level lower than one percent would mean slightly fewer hierarchical influence structures but an unchanged general rank-order on the measures of control and influence. This means that setting the cutoff at one percent results in a somewhat conservative estimate of the local influence of the United States.

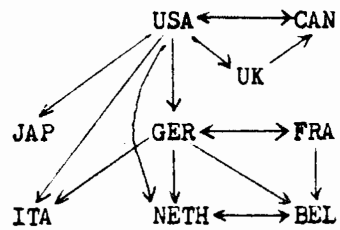
In contrast with the import influence structures for the Western Hemisphere, the import influence structures for the major North Atlantic nations and Japan are much more connected and less hierarchical. The structures in Figure 15 are for the year 1960—a threshold value of one percent of total imports is used on Figure 15a and a threshold of ten percent in Figure 15b. The trade data for these structures was collected by the OECD and published in Adams, Eguchi and Meyer-zu-Schlochtern (1969, p. 11). At the lower threshold, this set of nations forms a decidedly non-hierarchical influence structure. There are two main cliques: (1) the United States, Canada, Britain, and Japan, and (2) the EEC nations. The EEC nations, except for Germany, do not have import influence over Japan or vice versa. The only non-symmetrical influence relation is that between Japan and Germany.

In Figure 15b, however, a more hierarchical structure emerges. The United States, as usual, is a source and is interdependent only with Canada. The structure would be a hishinter except for the United States-Britain-Canada cycle of influence. If these three countries are assigned to the first level of the structure, then there are three levels: (1) US, UK, CAN, (2) JAP, GER, FRA, and (3) ITA, BEL, NETH. The United States has only indirect import influence over France and Belgium—possibly a reflection of the somewhat strained relations due to the Congo crisis and de Gaulle's policies—while it has direct influence over all the others. In any case, the North Atlantic nations and Japan form a much less hierarchical system than that formed by Western Hemisphere nations. Is it a coincidence that the former are wealthier and more powerful than the latter?

Other international influence structures can be observed by looking at arms transfers between nations. A particularly vivid pattern of influence structures results from the sale and purchase of guided missiles. In order to derive an influence



a. Threshold for import influence=1% of total imports



b. Threshold for import influence=10% of total imports

Abbreviations Used: BEL=Belgium; CAN=Canada, FRA=France, GER=West Germany, ITA=Italy, JAP=Japan, NETH=Netherlands, UK=Britain, and USA=United States.

FIGURE 15 Import influence structures for the North Atlantic and Japan.

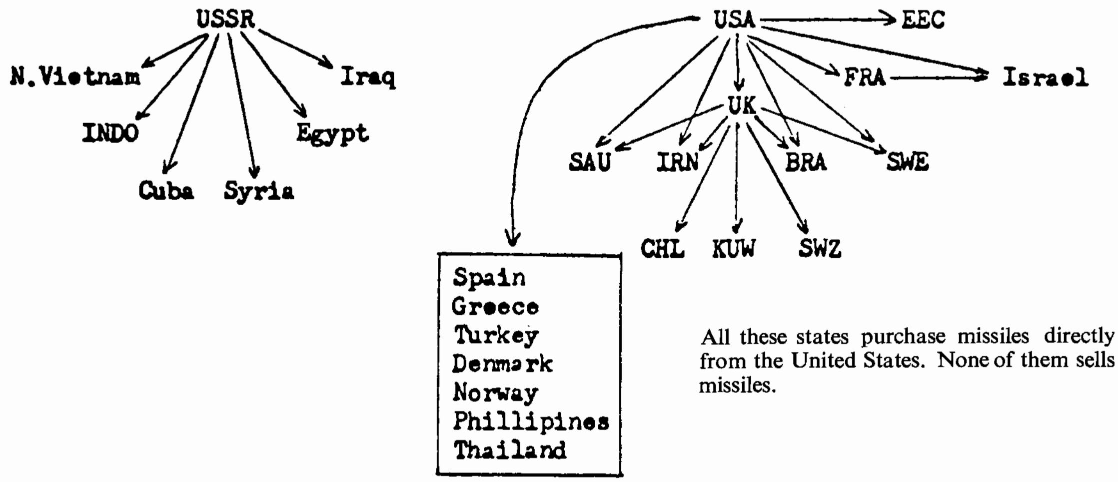
structure from transfers of missiles, assume that nation *i* influences nation *j*, if *i* sells missiles to *j* (*j* cannot produce the missiles domestically). Information on missile sales in the past decade comes from Cantori and Spiegel (1970, pp. 412–15). This truly global influence structure (see Figure 15) depicts all known missile sales during the period. The graph shows a strictly bipolar structure (disconnected with two components) containing a simple star-shaped tree (USSR) and a semilattice (US)—a hierarchical structure. We can easily determine local measures of influence without computation.

In addition to missile sales, conventional arms sales, exports, trade of certain commodities (such as manufactures or petroleum products), foreign investments, bilateral foreign aid, student exchanges, and tourism create interesting influence structures. Events data can yield measurement of potential or attempted influence and compliance in interactions between states.

Given the possibility of multiple domains of influence, how similar are influence structures for different domains? In the influence structures above, for example, the United States has import influence over many nations to whom it does not sell guided missiles. A nation may even hold influence over another in one domain but depend upon it in another domain. The Western Europeans, for example, depend on the Middle East for oil imports while the Middle East depends on imports of manufactured goods from Europe and the United States. Thus Arab states hold powerful bargaining power over the Europeans—a source of much international tension. A rotating leadership arrangement or shifting coalitions of voters or nations provoke similar tensions.

A *composite* influence structure results from the addition of different influence structures for each domain. A simple example of a composite influence structure is given in Figure 17.

One might expect inconsistent influence structures to foster greater systematic conflict or dissatisfaction than consistent structures, but this depends on the inconsistencies between local measures of influence and the form of the composite influence



Abbreviations Used: BRA=Brazil; CHL=Chile; EEC=West Germany, Italy, Belgium, Netherlands, and Luxembourg; FRA=France; INDO=Indonesia; IRN=Iran; KUW=Kuwait; SAU=Saudi Arabia; SWE=Sweden; SWZ=Switzerland; UK=United Kingdom; USA=United States; and USSR=Soviet Union.

FIGURE 16 International influence structure, missile sales, *circa* 1960–1970.

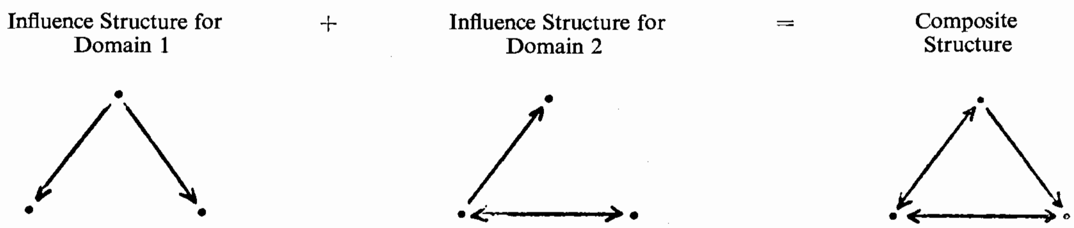


FIGURE 17 An example of a composite influence structure.

structure. A more connected and less hierarchical composite structure will more likely see its conflicts resolved by bargaining; and attitudes of units toward the system will be more positive.[3]

By contrast, contemporary international influence structures tend to be hierarchical and sometimes disconnected. Therefore, international actors ought to search for or create connected and non-hierarchical domains of influence. On a somewhat less systematic level, there appears to be some connection between cooperation and mutual influence and between conflict and moderate dependency levels. At higher levels of hierarchical influence structures, the most conflictive relations are experienced by disconnected sources or transmitters of influence—the bloc leaders.

It is hard to say whether the absence of influence or the high level of conflict is the cause or effect. In any case, there are clearly relationships between structures of influence and cooperation-conflict to explore.

STRUCTURES OF COOPERATION-CONFLICT

Cooperation-conflict is a concept increasingly used in quantitative studies of international politics to denote the amity-enmity of relations between pairs or sets of nations. The *signed* digraph, invented to deal with liking-disliking or amity-enmity relations in psychology, can be applied to international relations. The lines of a signed digraph can take on one of two values—either positive or negative. Positive lines are drawn just like regular lines in digraphs and negative lines are dotted (see Figure 18).

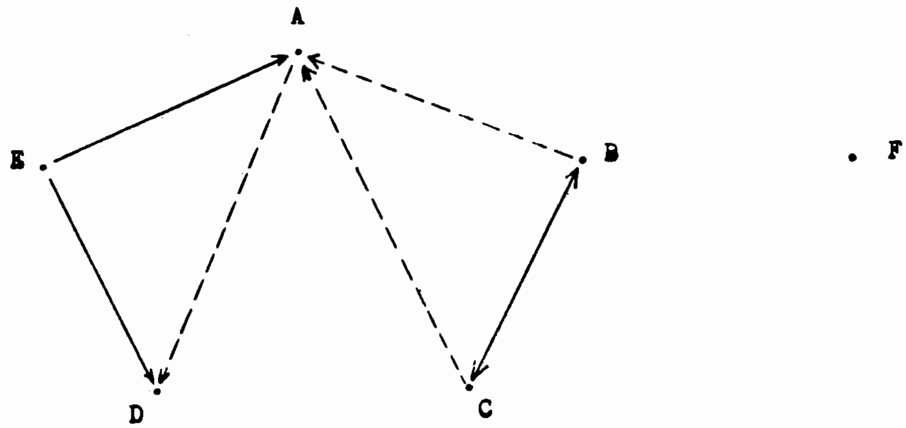


FIGURE 18 An example of a signed digraph.

The sign of a path, semipath, cycle or semicycle is positive if it has an even number of negative lines and negative if it has an odd number of negative lines. The semicycle from B to C to A and back is positive in Figure 17 because it has two negative lines.

Consider the following rules for amity and enmity:

a) A friend of a friend will be a friend; or at least not an enemy

b) An enemy of a friend will be an enemy; or at least not a friend

In Figure 19a, actor D is friendly with both E and F, while E and F are neither friends nor enemies. In Figure 19b, actor D is still friendly with E and F, but E decides he doesn't like F. At this point, D will probably feel somewhat uncertain about what he should do, preferring that his friends either liked each other or at least were not enemies. In Figure 19c, D decides to dislike F in order to preserve his friendship with E—a potentially stable situation. In Figure 19d, D decides that since E destroyed his friendship with F he doesn't like E any more. Even though this graph is unbalanced, such a situation is

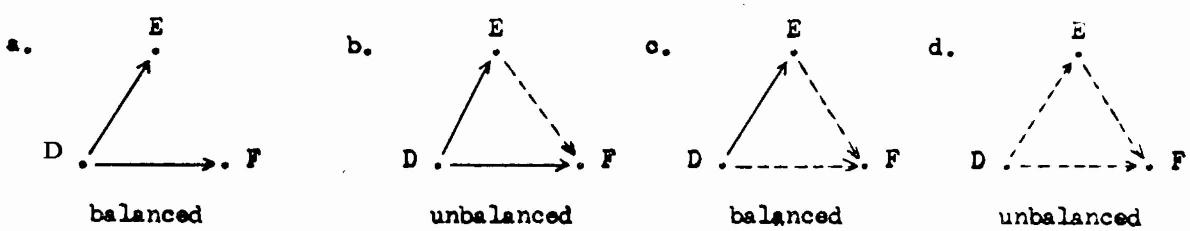


FIGURE 19 Examples of balanced and unbalanced signed digraphs.

c) A friend of an enemy will be an enemy; or at least not a friend

d) An enemy of an enemy will be a friend; or at least not an enemy.

These four rules taken together require a signed digraph representing amity and enmity to have no negative semi-cycles. This is the definition of *balance* in signed digraphs. The following patterns of amity-enmity in a 3-actor group demonstrate the intuitive nature of this requirement (Figure 19).

quite conceivable since the only rule of amity-enmity violated is rule (d). Rule (d) involves an implicit assumption that actors will form coalitions against common enemies, even at the expense of " letting bygones be bygones " with previous enemies. Sociologist James Davis (1967) relaxed this assumption; requiring only rules (a) and (c) results in the *clusterability* concept. A signed digraph is *clusterable* if it has no semicycles with only one negative line (e.g., Figure 19d). It follows from the definitions of balance and clusterability that if a signed digraph is balanced it is also clusterable but that if it is clusterable it is not necessarily balanced. Both balance and clusterability include digraphs with no negative lines; therefore, a unipolar international system is balanced and clusterable.

Several interesting mathematical properties of balance and clusterability point out other correspondences between the notions of international polarization and mathematical polarization. Mathematician Frank Harary (1953) first showed that if a digraph is balanced, the set of points can be partitioned into two subsets (one of which may be empty) so that any line joining points within the same subset is positive while any line joining two points in different subsets is negative (see Figure 20a—the two subsets are circled).

Davis (1967) suggested and Riley (1969) demonstrated that a clusterable signed digraph has the analogous property that its set of points can be partioned into subsets, called plus-sets, so that every positive line joins points in the same plus-set and every negative line joins points in different plus-sets (see Figure 20b).

Let an international pattern of cooperation-conflict be represented by a signed digraph thus: (1) if the level of cooperation-conflict exceeds a certain threshold value for a given directed dyad, draw a positive line from the point which represents the initiator to the point which represents the target; (2) if the level of cooperation-conflict is approximately neutral, draw no line; and (3) if the cooperation-conflict is lower than a threshold value for conflict, draw a negative line. If such a signed digraph is balanced, then the international system is bipolar or unipolar with respect to the threshold values. The system is unipolar if and only if all the lines are positive. If the signed digraph is clusterable but not balanced, then the international system must be multipolar.

Several important questions remain: Should neutral actors (such as point F in Figure 18) be excluded from the analysis, grouped with one of the poles or considered to constitute poles by themselves? If members of a given plus-set are antipathetic toward an opposing plus-set but within the plus-set there are even further subdivisions into disconnected cliques, should the plus-set be considered a single pole or several poles? (See Figure 21).

How many poles exist and which actors belong to them even when the signed digraph is neither balanced nor clusterable? Finally, how unbalanced or unclusterable is a system, especially in the case of some connection between the degree of polarization and the level of systemic conflict?

We assume here that unless a neutral actor is a dominant power (or hegemonic power), the neutral actor should not count as a separate pole. If a plus-set or subset contains disconnected cliques. however, and these cliques contain major powers, the number of poles will equal the number of cliques. Thus, the international system in Figure 8 would have four poles even though there are only three plus-sets. The question of the degree of polarization and the composition of poles when systems are not balanced or clusterable is somewhat more complex. Cartwright, Harary and Norman (1965) discuss several indices for the degree of balance including:

β = the number of positive semicycles in the digraph divided by the total number of semicycles,

a. A balanced signed digraph

b. A clusterable signed digraph

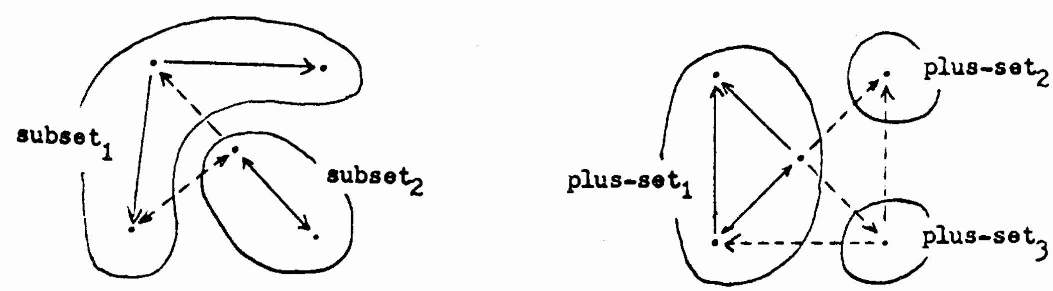


FIGURE 20 Examples of balanced and clusterable signed digraphs.

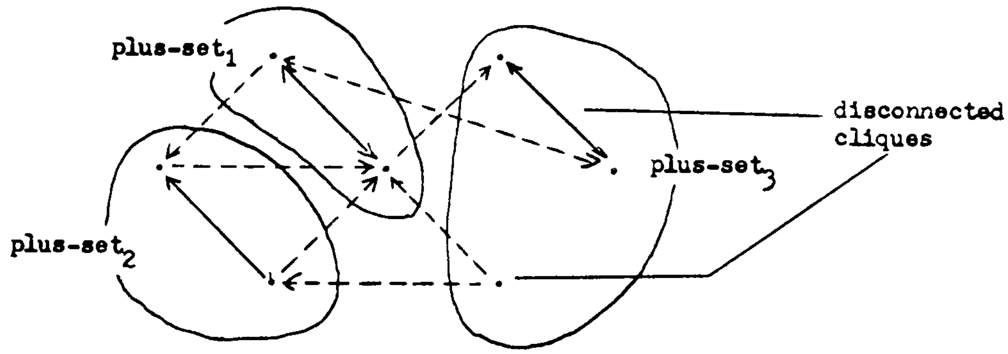


FIGURE 21 Example of a clusterable digraph with a disconnected plus-set.

and

λ = the minimal number of lines which must be changed (removed or sign changed) before the digraph is balanced.

In the example in Figure 18, there are four semicycles (A-E-D-A, A-B-C-A, A-C-B-A, and B-C-B). All but one are positive, so $\beta = 0.75$. Only one line must be changed or removed in order to make the digraph balanced, the line from A to D, so $\lambda = 1$. Analogous indices for the degree of clusterability would be:

γ = the number of semicycles without a single negative line divided by the total number of semicycles,

and

μ = the minimal number of lines which must be changed before the digraph is clusterable.

Clearly γ must be greater than or equal to β and μ must be less than or equal to λ for any signed digraph. $\beta = \gamma$ if and only if there are no negative semicycles with more than one negative line.

Peter Abell (1968) suggests that it is not necessary to consider all the semicycles in a directed graph to measure imbalance because: (1) the sociological justification for the use of semicycles of length greater than three is much weaker than that for semicycles of length two and three; (2) the measure of imbalance which considers only semicycles of length two or three,

$\beta_{2,3}$ = the number of positive semicycles of length two or three divided by the total number of semicycles of length two or three,

is much easier to compute than β; and (3) β and $\beta_{2,3}$ increase monotonically with one another (and are, therefore, perfectly correlated). Thus, in the analysis below, we use $\beta_{2,3}$ and $\gamma_{2,3}$ to measure the degree of polarization. We also use them to identify types of international systems using the following criteria:

Type of system	$\beta_{2,3}$	$\gamma_{2,3}$	Additional criteria
Strict unipolar	Equal to 1	Equal to 1	All lines are positive, no disconnected cliques
Loose unipolar	Less than 1	Equal to $\beta_{2,3}$	Number of negative lines less than or equal to λ
Strict bipolar	Equal to 1	Equal to 1	At least one negative line and no disconnected cliques or no negative line and two disconnected cliques
Loose bipolar	Less than 1	Equal to $\beta_{2,3}$	Number of negative lines greater than λ
Strict multipolar	Less than 1	Equal to 1	None
Loose multipolar	Less than 1	Greater than or equal to $\beta_{2,3}$	If $\gamma_{2,3} = \beta_{2,3}$ then there must be at least two disconnected cliques in one of the subsets

It is easy to identify the number and composition of poles when the signed digraph is balanced or clusterable (see Figures 20 and 21). This is somewhat more difficult with unbalanced or unclusterable digraphs in which there may be a number of ways of balancing or clustering the points according to various criteria (Flament, 1963). For example, in Figure 18 above, only one line must be changed to balance the digraph—but which line is changed makes a great difference in estimating the membership of poles. If line AD is removed or changed the digraph would be balanced, the system would be bipolar and the poles would be A, E, and D versus B and C. If line EA is removed or changed, the digraph would be balanced, the system tripolar and the poles E and D versus A versus B and C. One might, in such a case, refer to the strength of the relationships between A and D and between E and A in order to decide which would be most likely to change.

The probability that cooperation-conflict structures will be perfectly polarized is quite low if we assume that cooperative and conflictive relations are randomly distributed and the structure highly interconnected. Nevertheless, such structures do, in fact, tend toward polarization—and bipolarization, in particular. In a paper on the European powers during the 1870s (Hart, 1972), I found a tendency toward balance. A stronger tendency toward balance is evident in the data in Frank Klingberg's (1961) study of relations among major powers prior to World War II. I think it safe to assume that the major powers prior to World War I were bipolarized and that the cooperation-conflict structure for the nations in Figure 16 is almost perfectly balanced.

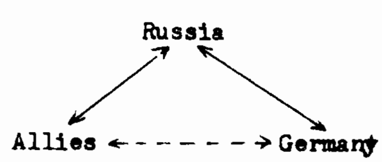
Despite this tendency toward balance there are many interesting examples of imbalance in international subsystems. Alan Dowty (1970, p. 98) gives an example of a clusterable structure in India, circa 1400–1500, when the states of Malwa, Mewar, Gujarat and Delhi form a tripolarized system (Delhi is allied with Gujarat). In Klingberg's 1961 study, the conclusion of the Russo-German Non-Aggression Pact of 1939 resulted in unbalanced structure since Russia had previously aligned herself with the Allies. Examples of imbalance can be seen in recent events such as the United States' policy change toward China and India's support of Bangladesh against Pakistan (see Figure 22). The new China policy put some strain on the détente between the United States and the Soviet Union while the Bangladesh incident had a strong effect on India's American and Indian-Soviet relations.

Cooperation-conflict structures help to quantify the degree of polarization in international systems with respect to enmity and alignment and to classify systems according to a typology which has great theoretical interest. They can clarify the differences between systems which are merely nonconflictive and those in which cooperation is widespread—sometimes called negative and positive peace. Like influence structures, they allow the researcher to supplement the traditional attributive approaches with relational or structural approaches.

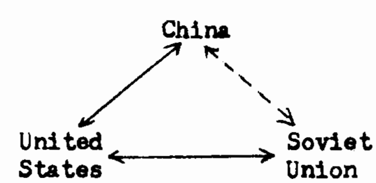
SUPERIMPOSITION OF STRUCTURE OF INFLUENCE AND COOPERATION-CONFLICT

An immediate consequence of considering influence and cooperation-conflict simultaneously is that of determining whether the type of polarization coincides. For example, if the structure is bipolar with respect to influence, is it also bipolar with respect to cooperation-conflict? Are the poles or blocs formed by the influence structure the same as those formed by the cooperation-conflict structure? Consider the simple case of tightly polarized

(a) The Russo-German Non-Aggression Pact.



(b) The *Détente* between China and the United States.



(c) The India-Bangladesh Incident.

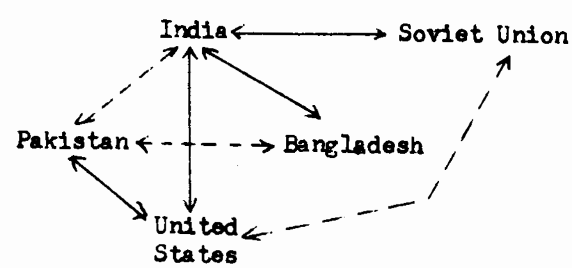


FIGURE 22 Examples of imbalance in international relations.

structures. If the poles are to overlap perfectly, then the following propositions must hold:

1) There can be cooperative relations only where there are direct or indirect influence relations and conflictive relations only between actors which cannot directly or indirectly influence one another

2) There can never be cooperative relations between sources or transmitters of disconnected influence substructures

3) Any conflict between actors on different levels must go from one connected influence substructure to another.

The first proposition implies that systemic cooperation is somewhat related to the connectivity of the influence structure. A disconnected structure limits the amount of cooperation to that which can be contained in the connected substructures. This is not unlike the argument that the increasing " interdependence " of nations in the contemporary era is likely to result in better prospects for world peace. But the difference is that only certain kinds of interdependence—i.e., those which link together previously disconnected substructures—can significantly effect cooperation levels. The second proposition rules out the possibility of détente or of integration efforts on the part of major powers. Thus, it is inconsistent for a major power to maintain the goals of bloc solidarity and détente at the same time. The third proposition means that the pursuit of bloc solidarity by major powers in hierarchical structures will make it difficult for minor powers to alter their relations with major powers without becoming neutral or shifting to another pole in the system. All are intuitively reasonable propositions about polarized systems and illustrate the utility of considering several types of structures simultaneously.

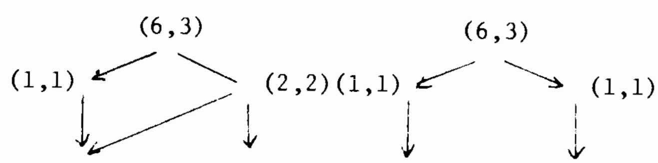
But it is not necessary to assume that polarization with respect to influence resembles that resulting from patterns of cooperation-conflict. There is some evidence, for example, that the European system of the 1870s was bipolarized with respect to cooperation-conflict, but multipolar in terms of influence. Inconsistent patterns of polarization may be necessary for furthering prospects for peace, just as loose polarization or decreasingly hierarchical influence structures may be conducive to peace.

CONCLUSION

Given the generality of the approach, the relative simplicity of graph theory, the dearth of structural investigations of international politics and the new research methods for obtaining data on international behavior, this appears to be a line of inquiry which may prove useful and enlightening. Influence structures indirectly measure the influence or power of social units—a way which takes into account the possibility that a unit with few power resources can improve its influence by various strategies: (a) specializing in influencing more powerful units, (b) cutting its dependency on more powerful units, or (c) involving itself in a clique or nonhierarchical substructure. Both influence and cooperation-conflict structures prove to be useful in making more precise the notion of polarization, a central concept in international relations theory. Finally, both kinds of structures can be used to observe and make predictions about small changes in the international environment which may have major consequences for international peace.

NOTES

1. Most of the graph theoretical definitions were taken from Cartwright, Harary and Norman (1965, pp. 404–10). The wordings of some of the definitions were taken from Davis (1972).
2. It is always necessary to remove lines to convert a semilattice into a tree. Thus, consider a " comparable " tree to be one which is formed by removing a minimal number of lines from the semilattice. Then it can be shown that the status and control of units on intermediate levels of the semilattice are always greater than or equal to that of corresponding units of the comparable tree. Below is a specific example. The numbers in parentheses are the status and control of each unit.



3. A clique is a minimally hierarchical and maximally connected influence structure.

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